Random Access Cooperative Communication

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Abstract—**User cooperation has been studied extensively in the literature. This mechanism achieves many of the same gains that can be had by using multi-antenna (MIMO) communications in applications where the size or computational resources of a node are limited. However, existing analysis assumes that the system is scheduled; both the relay and the destination know to listen when a packet is being sent. We present analysis of a class of random access cooperative systems and show that the same performance as the scheduled systems can be achieved as long as certain requirements are met in the packet detection scheme of the relay. In particular, we find that cooperative networks with a static decision threshold on energy detection perform asymptotically no better than simple point-to-point non-cooperative links. However, a decision threshold that dynamically shifts with average SNR allows the system to achieve full spatial diversity.**

Index Terms—**Random access, relay, cooperation, amplify-and-forward**

I. INTRODUCTION

User cooperation represents a class of techniques to exploit the transmission capabilities of nearby users to increase the reliability of a wireless communication link. The research area has grown from the original study of the relay channel by van der Meulen [1, 2], and later by Cover and El Gamal [3]. Foundational amplify-and-forward and decode-and-forward protocols for cooperation were proposed and analyzed by Laneman *et al.* in the context of delay-constrained quasi-static fading channels [4], and this work has served as the basis for many extensions [5–7]. A critical assumption in these prior works is that a cooperative relay must know *when* a source transmits, i.e. with perfect synchronization. In other words, these protocols and their analyses apply to the area of scheduled access systems. For this paper, we are interested in random access systems.

Our formulation, described in detail in Section II, asks the question of *when* bits are sent in addition to the canonical communication problem of *which* bits are sent. This additional source randomness necessitates a subsystem at a relay to enable "packet detection" to determine whether or not a source is sending. A practical design assumption in random access receivers is that the packet detector acts independently of average SNR. In this paper, we show that this assumption dominates the error performance of the system, effectively removing the diversity order benefits offered by user cooperation. We propose a scheme that dynamically adjusts with average SNR and show this scheme retains full diversity order.

A. Outline of Paper

Section II describes the system model and the error events that can occur. Section III considers a particular kind of packet detector at the relay in the form of an energy detector. Coupled with the results from the Section II, the likelihoods of error events as functions of average SNR are discussed. This analysis is general and can be used to analyze any relaying protocol that uses fixed slot durations (e.g. decode-and-forward). We then apply this general analysis to study one class of protocols, amplify-and-forward, in depth. Amplify-and-forward protocols are perhaps the simplest to implement, requiring only memory and a multiplier at the relay. Section IV analyzes the system performance conditioned on the aforementioned error events. This analysis coupled with the error event likelihoods culminates in analysis of overall system performance in Section V. We find that a static decision threshold on energy detection results in a cooperative network that performs asymptotically no better than a simple point-to-point network with no relay at all. However, a decision threshold that dynamically shifts with average SNR allows systems to achieve full cooperative diversity in a single-relay network, or

$$P_{\text{out}} \doteq \text{SNR}^{-2}, \qquad (1)$$

where P_{out} is the probability of outage (i.e. the probability of the instantaneous achievable rate falling below the transmission rate).[1]

II. SYSTEM MODEL

We assume that the source contends for the medium in frames, which we define to be two-slot units. In the first slot, it transmits with some average power (denoted by P), and it is silent in the second slot (denoted by 0). If the source has nothing to send, we assume that it remains silent over two slots. This assumption yields a special case of a transmitter that, in general, can be silent for any integer number of slots. We do not, however, exploit this analytical convenience at the relay. As such, we assume the relay is only slot-synchronized and has no notion of frames. The relay is assumed to not be synchronized with the source, and thus is forced to detect

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[1]The notation $\doteq$ means "equal in exponential order" and is used in the same way as [6].
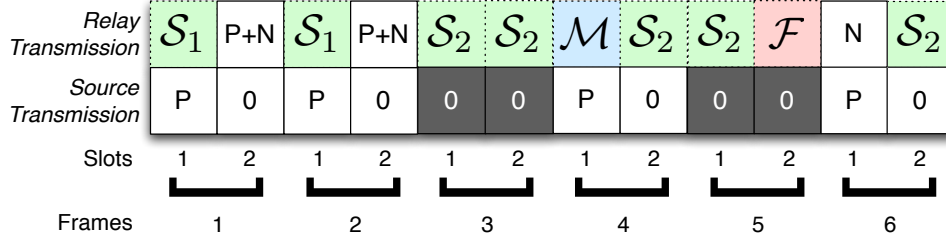
Fig. 1. Example sequence of relay detection events in system model

the presence or lack of the transmission. We assume a quasi-static fading model whose instantaneous channel conditions are constant for the duration of a single frame (i.e. two slots). This assumption aligns source channel usage with independent channel draws (i.e. the channel does not change between the first and second slot of a frame). This ensures that any diversity order improvements in the system are spatial and not temporal.

Under our random access framework, the task of a relay is not merely to amplify-and-forward what it receives, but also to decide whether or not something was sent in the first place. To describe the possible error events in this scenario, we introduce a slotted random access network model. The relay's behavior can be described by the following rules:

- If a packet is detected in the previous slot, amplify-and-forward the received waveform in the current slot.
- If no packet was detected in the previous slot, continue sensing the medium in the current slot.

As shown in Figure 2, we can define four possible packet detection events at the relay. Successfully detecting the presence of a packet (P↦P) corresponds to event $\mathcal{S}_1$. Similarly, successfully ignoring a slot when no transmit occurs (0↦0) corresponds to event $\mathcal{S}_2$. However, in the event that a transmitted codeword is missed (P↦0), the missed detection event is labeled $\mathcal{M}$. Finally, falsely identifying noise as a transmission (0↦P) is deemed false alarm $\mathcal{F}$. We use the notation $P_{\mathcal{M}}$ and $P_{\mathcal{F}}$, respectively, to describe the probability of error events $\mathcal{M}$ and $\mathcal{F}$. Additionally, we use the notation q to describe the source transmission probability.
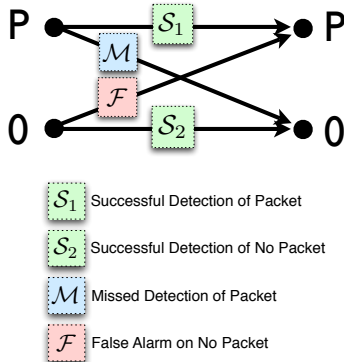
In Figure 1, we show a potential sequence of events in this system. The figure shows the transmits and lack thereof by the source and relay. Per the behavioral description earlier, events $\mathcal{S}_1$ and $\mathcal{F}$ trigger action at the relay. For either of these events, the relay transmits an amplified stored waveform in the following slot. Depending on what waveform was *actually* transmitted, we externally apply labels to the relay's waveform. If event $\mathcal{S}_1$ occurs at the beginning of a transmission slot, the transmitted waveform from the relay is denoted as $P+N$, representing the fact that the slot contains both source data and noise. If event $\mathcal{F}$ occurs, the waveform transmitted by the relay is amplified noise and is represented by N. The sequence in Figure 1 shows four "busy" frames (one, two, four, and six) and two "idle" frames (three and five). Of the busy frames, we can see that the relay behaves perfectly in frames one and two by successfully detecting the presence of a source transmission in the first slot and forwarding the received waveform in the second slot. However, in the fourth frame, the relay never transmits because it missed the detection of the source's transmission in the first slot. In the sixth frame, the relay actively hurts the source's ability to communicate by transmitting noise during the first slot of the frame because it falsely believed the source transmitted in the second slot of the previous frame.
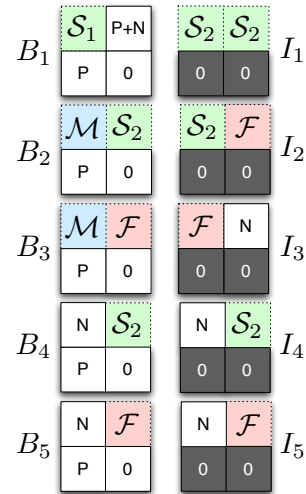

Fig. 2. Possible relay detection events

Fig. 3. Network states in slotted random access model. The bottom rows of blocks represent source activity and the top rows of blocks represent relay activity.

For any given channel draw (i.e. a given frame), any one of ten network states shown in Figure 3 is possible. States B_1 to B_5 represent the source "busy" states, where the source is actively transmitting. Similarly, states I_1 to I_5 represent the source "idle" states. We further categorize the busy states to describe their importance from the perspective of the destination node trying to decode the packet.

- **Best Case:** B_1 represents the case where the relay behaves perfectly. The relay actively helps the source communicate by forwarding its received waveform in the second slot of the frame.
- **Neutral Cases:** B_2 and B_3 represent cases where the relay neither helps nor hurts the source. In both slots of the frame, the relay senses the medium, and thus, never transmits due to a half-duplex constraint.
- **Worst Cases:** B_4 and B_5 represent cases where the relay actively hurts the source's ability to communicate by transmitting noise during the first slot of the frame.

Each of these states has some likelihood of occurring that can be computed by considering the probabilities of the *transitions* between these states and treating the system as a Markov chain. For example, B_1 can transition to I_2 if the source is idle in the next frame ($\bar{q}$), the first slot of noise does not result in a false alarm ($\bar{P}_{\mathcal{F}}$), and the second slot does result in a false alarm ($P_{\mathcal{F}}$). When conditioned on a particular channel gain, these probabilities arise from random noise. Hence, the probability of this transition occurring is their product. By considering all possible transitions to and from all possible states, steady-state likelihoods can be calculated for each busy state [8], yielding

$$P_{B_1} = \frac{(1 - P_{\mathcal{F}})(1 - P_{\mathcal{M}})q}{qP_{\mathcal{F}}^2 - P_{\mathcal{F}}^2 + P_{\mathcal{M}}qP_{\mathcal{F}} - qP_{\mathcal{F}} + 1} \quad (2)$$

$$P_{B_2} = \frac{(1 - P_{\mathcal{F}})^2 P_{\mathcal{M}}q}{qP_{\mathcal{F}}^2 - P_{\mathcal{F}}^2 + P_{\mathcal{M}}qP_{\mathcal{F}} - qP_{\mathcal{F}} + 1} \quad (3)$$

$$P_{B_3} = \frac{(1 - P_{\mathcal{F}})P_{\mathcal{F}}P_{\mathcal{M}}q}{qP_{\mathcal{F}}^2 - P_{\mathcal{F}}^2 + P_{\mathcal{M}}qP_{\mathcal{F}} - qP_{\mathcal{F}} + 1} \quad (4)$$

$$P_{B_4} = \frac{(1 - P_{\mathcal{F}})P_{\mathcal{F}}q\left(1 - (P_{\mathcal{F}}(1 - q) + (1 - P_{\mathcal{M}})q)\right)}{qP_{\mathcal{F}}^2 - P_{\mathcal{F}}^2 + P_{\mathcal{M}}qP_{\mathcal{F}} - qP_{\mathcal{F}} + 1} \quad (5)$$

$$P_{B_5} = \frac{P_{\mathcal{F}}^2 q\left(1 - (P_{\mathcal{F}}(1 - q) + (1 - P_{\mathcal{M}})q)\right)}{qP_{\mathcal{F}}^2 - P_{\mathcal{F}}^2 + P_{\mathcal{M}}qP_{\mathcal{F}} - qP_{\mathcal{F}} + 1} \quad (6)$$

for $q \in (0, 1)$, $P_{\mathcal{M}} \in (0, 1)$, and $P_{\mathcal{F}} \in (0, 1)$.

III. ENERGY DETECTION

We now turn to the task of determining the relationship between the state likelihoods and average SNR. To accomplish this, we must determine how $P_{\mathcal{M}}$ and $P_{\mathcal{F}}$ themselves depend on average SNR. These relationships depend on the type of packet detection being employed. In the role of packet detection for amplify-and-forward networks, energy detection is intuitively satisfying. If a relay detects a large amount of energy, it can decide that the source is transmitting. If a small amount of energy is detected, it might decide that the energy is only due to noise. This process requires no knowledge of

the source waveform, which is a key benefit of amplify-and-forward systems.

Energy detection is a well-studied problem dating back to Urkowitz who studied the problem of detection of unknown, but deterministic, signals in AWGN channels [9]. In recent years, Digham has extended this work to a variety of different quasi-static fading channels [10, 11]. In this section, we relax the deterministic assumption in these prior works to consider energy detection of random signals. This relaxation aligns this analysis to the random-coding arguments implicit in Section IV.

We adopt the following system model and nomenclature. In particular, we model the signal received by the relay as

$$y_r[n] = h_{s,r}Mx_s[n] + z_r[n],$$

where $n \in \left[0, 1, 2, \ldots, \frac{L}{2}\right]$ (i.e. half of a frame), x_s is assumed to be a zero-mean, circularly symmetric Gaussian random vector of variance SNR, z_r is assumed to be a zero-mean, circularly symmetric Gaussian random vector of unit variance, and $h_{s,r}$ is the instantaneous quasi-static channel realization assumed to be constant for the duration of L. Under this formulation, SNR represents the source-relay average signal-to-noise ratio.

$$M = \begin{cases} 0 & \text{source is idle} \\ 1 & \text{source is busy} \end{cases} \quad (7)$$

represents the source's state. With this nomenclature in place, we can construct the statistic that will be used by the energy detector to make a hard decision on the presence of a packet or lack thereof. This decision is the energy of the vector, or equivalently, the sum of squared amplitudes of the vector elements. Formally,

$$S = \sum_{n=1}^{\frac{L}{2}} |\sqrt{2} \cdot y_r[n]|^2$$

where $\sqrt{2}$ is simply a scaling factor that eases the notation later in the analysis. From this point, we consider the distribution on S for one of two hypotheses: nothing is sent or something is sent. We represent the hypotheses by H_M, where M is the source activity defined in Equation (7). We can represent the distribution of S for either hypothesis with

$$S \sim \begin{cases} \chi_L^2(x) & H_0 \\ \frac{\chi_L^2\left(\frac{s}{1 + |h_{s,r}|^2 \text{SNR}}\right)}{1 + |h_{s,r}|^2 \text{SNR}} & H_1 \end{cases} \quad (8)$$

where χ_L^2 is a central chi-square distribution with L degrees of freedom. With these distributions, we are finally ready to assign functions relating the probabilities of the detection events shown in Figure 2 to SNR and $|h_{s,r}|^2$.

A. Probability of False Alarm

The false alarm event represents the case where a particular noise vector has "too much" energy. Formally,

$$P_{\mathcal{F}} = Pr\{S > \Lambda | H_0\},$$

where Λ is the decision threshold in the system. This expression is equivalent to the complement of the cumulative density function (CDF) of the distribution evaluated at Λ. Because this CDF is known for a central chi-square distribution, we can write the probability of a false alarm event as

$$P_{\mathcal{F}} = 1 - \frac{\gamma\left(\frac{L}{2}, \frac{\Lambda}{2}\right)}{\Gamma\left(\frac{L}{2}\right)} = 1 - \mathcal{P}\left(\frac{L}{2}, \frac{\Lambda}{2}\right), \tag{9}$$

where γ is the lower incomplete Gamma function, Γ is the Gamma function, and $\mathcal{P}$ is the regularized Gamma function. This expression relates the probability of false alarms to the decision threshold Λ and the codeword length L.

B. Probability of Missed Detection

The missed detection event represents the case where a particular channel realization suppresses the source's energy below the required threshold. Formally,

$$P_{\mathcal{M}} = Pr\{S < \Lambda | H_1\},$$

where Λ is the decision threshold in the system. This expression is the CDF of the distribution evaluated at a Λ that is scaled according to the received SNR at the relay. We write this missed detection probability as

$$P_{\mathcal{M}} = \mathcal{P}\left(\frac{L}{2}, \frac{\Lambda}{2\left(1 + |h_{s,r}|^2 \text{SNR}\right)}\right). \tag{10}$$

This expression relates the probability of missed detections to the decision threshold Λ, the codeword length L, the average SNR, and the instantaneous channel realization $h_{s,r}$.

C. Relationship of State Likelihoods to SNR

By considering energy detection as a suitable packet detector for the relay, we now have expressions for missed detection and false alarm probabilities that can be substituted into Equations (2-6). We consider two scenarios for the energy detector: a static threshold $\Lambda = K$ and a dynamic threshold $\Lambda = K_1 + K_2 \log(1 + \text{SNR})$.

1) Static Energy Detection Threshold: By substituting Equations (9) and (10) into Equations (2-6) and averaging over Rayleigh fading gains $|h_{s,r}|^2$, one finds that for static energy thresholds only certain states decay in likelihood as functions of SNR. Specifically,

$$P_{B_1} \doteq \text{SNR}^0 \tag{11}$$
$$P_{B_2} \doteq \text{SNR}^{-1} \tag{12}$$
$$P_{B_3} \doteq \text{SNR}^{-1} \tag{13}$$
$$P_{B_4} \doteq \text{SNR}^0 \tag{14}$$
$$P_{B_5} \doteq \text{SNR}^0. \tag{15}$$

The key observation here is that the worst case states B_4 and B_5 have no dependence on SNR. This arises from Equation (9), where false alarms occur with a fixed likelihood when a static energy threshold Λ is employed.

2) Dynamic Energy Detection Threshold: When a dynamic threshold $\Lambda = K_1 + K_2 \log(1 + \text{SNR})$ is employed, however, we see that all error states can decay with SNR. Specifically,

$$P_{B_1} \doteq \text{SNR}^0 \tag{16}$$
$$P_{B_2} \doteq \text{SNR}^{-1} \tag{17}$$
$$P_{B_3} \doteq \text{SNR}^{-1} \tag{18}$$
$$P_{B_4} \doteq \text{SNR}^{-C} \tag{19}$$
$$P_{B_5} \doteq \text{SNR}^{-C} \tag{20}$$

where C is a constant that depends on K_1 and K_2. The exact nature of this dependence can be determined but is not important to this particular work [8]; the key observation is that the worst case states B_4 and B_5 can be made to decay in likelihood arbitrarily fast with SNR.

IV. CONDITIONAL PERFORMANCE

Having established the likelihoods of the best, neutral, and worst states in the network, we now turn to describing the performance of the network conditioned on each of these states. The performance metric we will use is diversity order (i.e. the rate of decay of outage probability versus SNR on a logarithmic scale). For proofs, the reader is referred to [8].

A. Best Case State

The best case state B_1 captures the event when the relay actively helps the source communicate to the destination. This event is the same event that occurs for all time in the scheduled access amplify-and-forward system originally studied in [4]. As such, the diversity order in this case is known to be two, or

$$P_{\text{out}|B_1} \doteq \text{SNR}^{-2}. \tag{21}$$

B. Neutral Case States

The neutral case states B_2 and B_3 capture the events when the relay is effectively disabled, neither helping nor hurting the source-destination link. Thus, the diversity for these cases is the same as the diversity order for a point-to-point SISO link. As such, the diversity order in these cases is known to be one, or

$$P_{\text{out}|B_2} \doteq \text{SNR}^{-1} \tag{22}$$
$$P_{\text{out}|B_3} \doteq \text{SNR}^{-1}. \tag{23}$$

C. Worst Case States

The worst case states B_4 and B_5 capture the events when the relay actively impedes the source's ability to communicate by transmitting noise at the same time. At best, these cases performs no better than than the neutral cases. Hence, we can loosely bound the diversity order of the worst case states between zero and one, or

$$\mathrm{SNR}^0 \mathrel{\dot{\geq}} P_{\mathrm{out}|B_4} \mathrel{\dot{\geq}} \mathrm{SNR}^{-1} \qquad (24)$$

$$\mathrm{SNR}^0 \mathrel{\dot{\geq}} P_{\mathrm{out}|B_5} \mathrel{\dot{\geq}} \mathrm{SNR}^{-1}. \qquad (25)$$

V. Total Performance

We finally turn to discussing the total performance of the network by combining the state likelihoods with conditional diversity orders via the law of total probability

$$P_{\mathrm{out}} = \sum_{i=1}^{5} P_{\mathrm{out}|B_i} P_{B_i} \qquad (26)$$

where the P_{B_i} terms are from Section III and the $P_{\mathrm{out}|B_i}$ terms are from Section IV.

A. Static Energy Detection Threshold

Substituting Equations (11-15) and (21-25) into Equation (26) yields

$$\mathrm{SNR}^0 \mathrel{\dot{\geq}} P_{\mathrm{out}} \mathrel{\dot{\geq}} \mathrm{SNR}^{-1}. \qquad (27)$$

This says that, with a static energy threshold, the diversity order of the system is, at best, one. Thus, full cooperative diversity is not achieved in this scheme.
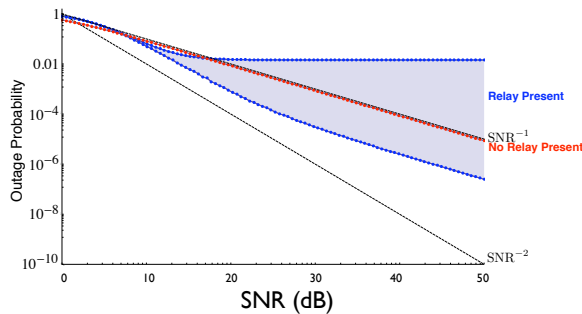

Fig. 4. Simulation with static energy detection threshold

Figure 4 shows a Monte-Carlo simulation of the random access cooperative network with a static energy threshold employed at the relay. From a diversity order standpoint, the network performs no better than having no relay at all.

B. Dynamic Energy Detection Threshold

Substituting Equations (16-20) and (21-25) into Equation (26) yields

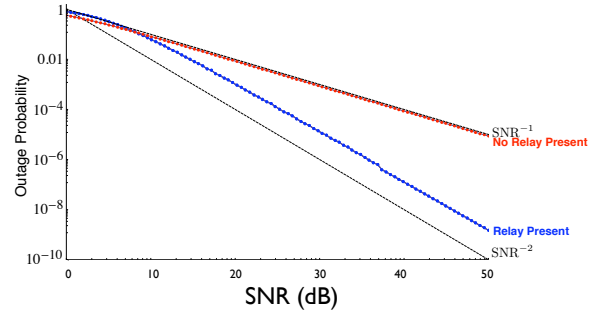$$P_{\mathrm{out}} \mathrel{\dot{=}} \mathrm{SNR}^{-2}. \qquad (28)$$

Fig. 5. Simulation with dynamic energy detection threshold

With a dynamic energy threshold, the full diversity from Equation (1) is retained.

Figure 5 shows a Monte-Carlo simulation with a dynamic threshold employed at the relay. Full diversity of two is maintained in the random access cooperative network.

VI. Conclusions

In this work, we have shown that the energy detection decision made at the relay to address random access can greatly impact the diversity order of a cooperative system. Specifically, if this decision is made independently of SNR, the diversity order of the overall cooperative link is no better than the standard point-to-point noncooperative diversity order. However, if this decision is made to be dependent upon SNR (i.e. dynamic), full cooperative diversity can be retained.

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