Understanding the Impact of Phase Noise on Active Cancellation in Wireless Full-Duplex

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Abstract—**Practical designs of wireless full-duplex are made feasible by reducing self-interference via active and passive methods. However, extending the range to long-range communication remains a challenge, primarily due to residual self-interference even after a combination of active cancellation and passive suppression methods is employed. In this paper, we study the factor that limits the amount of active cancellation in current designs of full-duplex. Through an experiment, we show that phase noise in the local oscillator limits the amount of active cancellation of the self-interference signal. Analysing the design proposed by [1, 2] in detail, we show that modifying the quality of the local oscillator can significantly increase the amount of active cancellation in full-duplex systems.**

I. Introduction

Full-duplex is the modality of communication where a node can transmit one signal and receive another signal simultaneously on the same frequency band. Due to simultaneous transmission and reception, the signal being transmitted interferes with the signal of interest being received at the full-duplex node. Such interference is called *self-interference*. Due to physical proximity of transmit and receive antenna on the full-duplex node, the self-interference is several magnitudes stronger than the signal of interest. The main challenge in enabling full-duplex is to manage the self-interference in such a way that ensures reliable decoding of the signal of interest.

In practical designs of full-duplex [1–9], self-interference is managed by reducing it by a combination of passive and active techniques. Passive methods rely on increasing the pathloss between the transmitter and receiver on the full-duplex node. Active cancellation techniques employ the knowledge of self-interference and inject a cancelling signal to create a null for the self-interference signal. However, experimental observations report that practical designs to date do not eliminate the self-interference completely. In fact [9] reports that strength of the residual self-interference even after passive and active cancellation is used is about 15 dB higher than the thermal noise floor. In this paper, our focus will be to understand the factors that limit active cancellation techniques from completely nullifying the self-interference.

The active cancellation method which cancels the self-interference prior to digitization of the received signal is called active analog cancellation. The active cancellation method that cancels the received signal after digitization is called digital cancellation. The limitation in active cancellation is highlighted by two experimental observations. First observation: In [1], the amount of active analog cancellation is limited to 35 dB, despite the fact the active analog canceller in [1] does not manage to bring the self-interference close to thermal noise floor after cancellation which begs the question *"Why is the amount of active analog cancellation in [1] limited?"*

Second observation: In [2], it is shown that when digital cancellation is serially concatenated with active analog cancellation, then the amount of digital cancellation deteriorates as the amount of active analog cancellation increases. More specifically, the sum total of the amount of active analog cancellation and digital cancellation is less than 35 dB, despite the fact the self-interference after digital cancellation is not close to the thermal noise floor. Thus, raising the second question *"Why does adding a digital cancellation stage in concatenation with active analog cancellation not achieve higher overall active cancellation?"*

In this paper, we answer both the questions, for the active canceller proposed and implemented by [1, 2] in two steps. First, via an experiment, we identify that phase noise in the local oscillator is a bottleneck that limits the amount of active cancellation. Through the controlled experiment, we show that even when the error in estimating the self-interference channel is neglibly small, phase noise causes perturbations due to which the self-interference signal and the cancelling signal do not perfectly null one another.

Secondly, we analyse the active analog canceller architecture proposed in [1, 2] by incorporating the pertubations due to phase noise. We compute the amount of active analog cancellation possible in [1, 2] in presence of phase noise and show that it closely matches the experimental observation reported in [1, 2], thus explaining the limitation in active analog cancellation. Further, we show the interdependence of digital and active analog cancellation by analysing the impact of digital cancellation on the uncanceled self-interference after active analog cancellation. Part of the uncanceled self-interference after active analog cancellation is due to phase noise, which continues to remain uncanceled even after digital cancellation because phase noise is independent of the self-interference signal. Thus, the sum total of digital and active analog cancellation in [2] is limited by phase noise of the local oscillator at the full-duplex node.

II. Reducing Self-Interference via Active Cancellation

The method of actively reducing self-interference that exploits the knowledge of the self-interference signal to inject a cancelling signal into the received signal before the received signal is digitized is called active analog cancellation. The method of actively cancelling the self-interference by injecting a cancelling signal after digitization of the received signal is referred to as digital cancellation.

In this Section, first we mathematically describe the received signal at the full-duplex node in the "conventional" sense and then analyse the limit of the amount of active cancellation due to estimation error. We show that estimation error does not satisfactorily explain limitations in active cancellation observed from experimental data in [1, 2].

A. Conventional System Model to Describe Self-Interference

In this paper, we will restrict our focus to narrowband SISO full-duplex systems. Let $\mathsf{N1}$ denote the full-duplex node and $\mathsf{N2}$ denote the node from which $\mathsf{N1}$ is receiving a signal-of-interest. Let $x_{\mathsf{si}}(t)$ and $x_{\mathsf{s}}(t)$ denote the self-interference signal and signal-of-interest. For a narrowband SISO channel, we can assume that the self-interference channel at node $\mathsf{N1}$ is modeled as a single delay tap channel and is denoted by $\mathbf{h}_{\mathsf{si}}(t) = h_{\mathsf{si}}\delta(t-\Delta_{\mathsf{si}})$, where $h_{\mathsf{si}} \in \mathbb{C}$ is attenuation and $\Delta_{\mathsf{si}} \in \mathbb{R}^+$ is the delay after which the self-interference signal $x_{\mathsf{si}}(t)$ arrives at the receiver. Similarly, the channel from the transmitter of $\mathsf{N2}$ to the receiver of $\mathsf{N1}$ is denoted by $\mathbf{h}_{\mathsf{s}}(t) = h_{\mathsf{s}}\delta(t-\Delta_{\mathsf{s}})$, where $h_{\mathsf{s}} \in \mathbb{C}$ and $\Delta_{\mathsf{s}} \in \mathbb{R}^+$. The received signal at node $\mathsf{N1}$, $y(t)$, is a combination of the self-interference signal and the signal-of-interest and is given by

$$y(t) \quad = \quad h_{\mathsf{si}}x_{\mathsf{si}}(t-\Delta_{\mathsf{si}}) + h_{\mathsf{s}}x_{\mathsf{s}}(t-\Delta_{\mathsf{s}}) + z(t), \quad (1)$$

where $z(t)$ is the receiver thermal noise with $\mathcal{N}(0,\sigma_z^2)$ distribution. The transmit power constraints at nodes $\mathsf{N1}$ and $\mathsf{N2}$ are described by

$$\mathbb{E}(|x_{\mathsf{si}}(t)|^2) \leq 1; \ \mathbb{E}(|x_{\mathsf{s}}(t)|^2) \leq 1. \quad (2)$$

B. Impact of Estimation Error on Active Cancellation

The objective of active cancellation is to create a null for the self-interference signal. Given an estimate of the self-interference channel, $\widehat{\mathbf{h}}_{\mathsf{si}}(t) = \hat{h}_{\mathsf{si}}\delta(t-\hat{\Delta}_{\mathsf{si}})$, a cancelling signal, $x_{\mathsf{c}}(t) = -\hat{h}_{\mathsf{si}}x_{\mathsf{si}}(t-\hat{\Delta}_{\mathsf{si}})$, can be generated. Adding the cancelling signal to the received self-interference signal results in a residual

$$y_{\mathsf{res}}(t) = h_{\mathsf{si}}x_{\mathsf{si}}(t-\Delta_{\mathsf{si}}) - \hat{h}_{\mathsf{si}}x_{\mathsf{si}}(t-\hat{\Delta}_{\mathsf{si}}) + z(t). \quad (3)$$

Let a training sequence $[s_1, s_2, \ldots, s_{\mathsf{train}}]$ of length T_{train}, with $\mathbb{E}(|s_i|^2) \leq 1$, where $i \in \{1, 2, \ldots, T_{\mathsf{train}}\}$ be used to obtain the estimate, $\widehat{\mathbf{h}}_{\mathsf{si}}(t)$, of the self-interference channel. Then, it can be shown

$$\mathbb{E}(|y_{\mathsf{res}}|^2) < \frac{5\sigma_z^2}{T_{\mathsf{train}}} + \sigma_z^2. \quad (4)$$

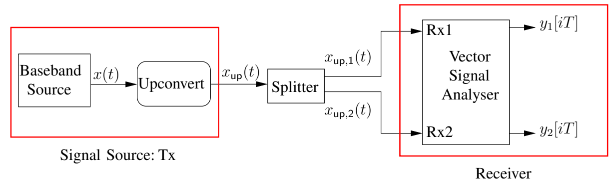

Fig. 1. Schematic of the experiment described in Section III-A

Detailed analysis is shown in the longer version [10]. According to (4), the residual should decay inversely with the length of training sequence and even with a very short training length, say $T_{\mathsf{train}} = 5$, the residual self-interference is no more than 3 dB above thermal noise. However, the observed phenomenon in [9], which uses the same design as [1, 2], is that the residual self-interference is 15 dB higher than the thermal noise which is clearly not explained by the signal model in (1), raising a possibility of some other source of impairment leading to a bottleneck in active cancellation.

III. Identifying Phase Noise as Bottleneck in Cancellation

In this section, we identify the bottleneck in active cancellation observed in [1, 2] by measuring the amount of active cancellation in a controlled experiment.

A. Experiment

The steps of the whose schematic is shown in Fig. 1 are:

- A signal $x(t) = e^{j\omega t}$ is digitally generated, with $\omega/2\pi$ =1MHz, and is upconverted to the carrier frequency of ω_c radians/sec at the transmitter Tx. Let $x_{\mathsf{up}}(t)$ denote the upconverted signal.
- The signal $x_{\mathsf{up}}(t)$ is split using a 3-port power splitter [11]. Let $x_{\mathsf{up},1}(t)$ and $x_{\mathsf{up},2}(t)$ denote the two signals output from the power splitter.
- Using a wired connection, the signals $x_{\mathsf{up},1}(t)$ and $x_{\mathsf{up},2}(t)$ are fed into two input ports of a vector signal analyzer (VSA) [12]. Let the two input ports denote the two receivers Rx1 and Rx2. Using the knowledge of ω_c, the VSA downconverts and digitizes the two received signals, which we denote as $y_1[iT]$ and $y_2[iT]$.

In the above experiment $i \in \mathbb{Z}$ and T is the sampling rate chosen to be 21.7 ns. The above experiment was conducted using two signal sources: an off-the-shelf radio chip [13] used in WARP [14] and a high precision Vector Signal Generator [15]. For WARP, $\omega_c/2\pi = 2.4$ GHz and for the Vector Signal Generator, $\omega_c/2\pi = 2.2$ GHz. In the above experiment, let Δ_1, Δ_2 denote the time taken for the signal to travel from Tx to the two receivers Rx1 and Rx2 respectively. The wires over which the signals travel were approximately of the same length, therefore $\Delta_1 \approx \Delta_2$.

B. Mimicking Active Cancellation

The transmitted signal, $x[iT]$, is narrowband. Therefore if the upconversion process does not add any noise, then the received sequences can be written as

$$y_1[iT] = h_1 e^{-j(\omega_c+\omega)\Delta_1} x[iT] + z_1[iT] \quad (5)$$
$$y_2[iT] = h_2 e^{-j(\omega_c+\omega)\Delta_2} x[iT] + z_2[iT], \quad (6)$$

where h_1 and h_2 are complex attenuations. Since the signals only travel over a wire, the attenuations h_1 and h_2 can be assumed to be constant. Note that, $y_1[iT]$ and $y_2[iT]$ are noisy versions of same signal, $x[iT]$, scaled by different quantities. Therefore, we can mimic active cancellation by subtracting a scaled and delayed version of the $y_1[iT]$ from $y_2[iT]$. The residual after active cancellation will be

$$y_{\mathsf{res},d}[iT] = y_2[iT] - h(d) y_1[(i-d)T], \quad (7)$$

where $d \in \mathbb{Z}$ is the delay in the number of samples, and $h(d)$ is the scaling as a function of the delay. Using $N \in \mathbb{N}$ samples of the received sequences $y_1[iT]$ and $y_2[iT]$, the scaling is computed as

$$h(d) = \frac{\sum_{i=1}^{N} y_2[iT]' y_1[(i-d)T]}{\sum_{i=1}^{N} |y_2[iT]|^2}. \quad (8)$$

If (5) and (6) hold true and if we rewrite $h(d) = \frac{h_1}{h_2} e^{j(\omega_c(\Delta_2-\Delta_1)+\omega dT)} + \epsilon$, then the expected strength of the residual signal is given by

$$\begin{aligned} & \mathbb{E}(|y_{\mathsf{res},d}[iT]|^2) \\ = & \mathbb{E}(|y_1[iT] - h(d) y_2[(i-d)T]|^2) \\ = & |h_2|^2 \mathbb{E}(|\epsilon|^2) + \mathbb{E}(|z_1[iT]|^2 + |z_2[(i-d)T]|^2) \\ = & |h_2|^2 \mathbb{E}(|\epsilon|^2) + 2\sigma_z^2. \end{aligned} \quad (9)$$

Further, it can be shown that by letting $N \to \infty$ we have

$$\mathbb{E}(|y_{\mathsf{res},d}[iT]|^2) = \frac{|h_1|^2}{|h_2|^2} \sigma_z^2 + 2\sigma_z^2. \quad (10)$$

For the experiment conducted $\frac{|h_1|^2}{|h_2|^2} \approx 1$, thus it is expected that strength of the residual self-interference should be approximately $3\sigma_z^2$, a quantity independent of the delay d.

C. Experiment: Results and their Explanation

In Fig. 2, we plot the amount of active cancellation as a function of delay d measured from the experiment for both the signal sources. For WARP as the signal source, the amount of active cancellation is a function of the delay. In contrast, for the Vector Signal Generator amount of active cancellation is roughly constant as delay varies.

a) Upper bound of cancellation: From Fig. 2, we note that for both the signal sources, the maximum active cancellation is around 55 dB. The limitation on the cancellation can be explained by the dynamic range of the measurement equipment. The data-sheet [12] of the VSA lists that it offers a dynamic range of anywhere between 55-60 dB. Thus, the received signals $y_1[iT]$ and $y_2[iT]$ themselves have an SNR of no more 55-60 dB, thereby limiting the maximum cancellation in the range of 55-60 dB only.

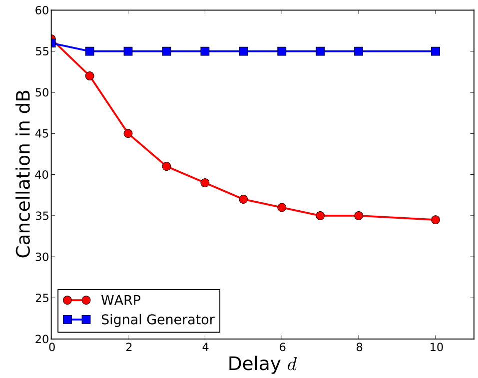

Fig. 2. Amount of cancellation as a function of the delay for different signal sources measured from the experiment. For WARP, even with delay $d = 100$, the amount of cancellation measured is ≈35 dB

b) Phase noise explains the trend of cancellation: The amount of cancellation measured, when WARP is used as a signal source, reduces as the delay increases and eventually settles down around 35 dB. Now, we will show that trend in amount of cancellation can be explained if we consider the presence of phase noise in the upconverted signal.

Phase noise is the jitter in the local oscillator. If the baseband signal $x(t)$ is upconverted to a carrier frequency of ω_c, then the upconverted signal $x_{\mathsf{up}}(t) = x(t)e^{j(\omega_c t + \phi(t))}$, where $\phi(t)$ represents the phase noise. While downconverting a signal, phase noise can be similarly defined. The variance of phase noise is defined as $\sigma_\phi^2 = \mathbb{E}(|\phi(t)|^2)$ and its autocorrelation function is denoted by $R_\phi(.)$. For a measurement equipment like VSA, the phase noise at the receiver is small. Therefore the total phase noise in the received signal, after downconversion, is dominated by phase noise at transmitter, i.e., the source of the signal. In presence of phase noise, the equations (5) and (6) can be rewritten as

$$y_1[iT] = h_1 e^{-j(\omega_c+\omega)\Delta_1} e^{j\phi[iT-\Delta_1]} x[iT] + z_1[iT],$$
$$y_2[iT] = h_2 e^{-j(\omega_c+\omega)\Delta_2} e^{j\phi[iT-\Delta_2]} x[iT] + z_2[iT].$$

For a delay d, suppose an oracle provides scaling $h(d) = \frac{h_1}{h_2} e^{j(\omega(\Delta_2-\Delta_1)+\omega dT)}$ to subtract a delayed version of $y_2[iT]$ from $y_1[iT]$, then the residual self-interference will be

$$\begin{aligned} & y_{\mathsf{res},d}[iT] \\ = & y_1[iT] - h(d) y_2[(i-d)T] \\ = & h_1 x[iT] e^{-j(\omega_c+\omega)\Delta_1} (e^{j\phi[iT-\Delta_1]} - e^{j\phi[iT-\Delta_2-dT]}) \\ & + z_1[iT] - z_2[(i-d)T] \\ \stackrel{(a)}{\approx} & j h_1 x[iT] e^{-j(\omega_c+\omega)\Delta_1} (\phi[iT-\Delta_1] - \phi[iT-\Delta_2-dT]) \\ & + z_1[iT] - z_2[(i-d)T], \end{aligned}$$

where (a) is valid if the phase noise is small. The resulting strength of the residual self-interference is

$$\mathbb{E}(|y_{\mathsf{res},d}[iT]|^2) \stackrel{(a)}{\approx} 2|h_1|^2 \sigma_\phi^2 (1 - R_\phi(dT)) + 2\sigma_z^2,$$

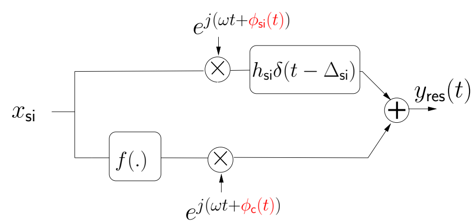

Fig. 3. Block diagram representation of the active analog canceller with used in [1, 2]

where the approximation (a) is reasonable since $\Delta_1 \approx \Delta_2$. As the delay increases, it is natural that the temporal correlation in phase noise reduces. Therefore the amount of cancellation, when WARP is used as a signal source, will reduce as the delay increases which explains the trend of cancellation in Fig. 2. Once the delay is sufficiently large, $R_\phi(dT) \approx 0$, thus the dependence of the residual on delay will vanish. For the MAXIM 2829 transceiver used in WARP, $\sigma_\phi \approx 0.7°$. For large delay d, $\sigma_\phi = 0.7°$ is equivalent to 35 dB cancellation which explains lower bound of cancellation. Although the trend in cancellation when the vector signal generator is used as the source does not appear to be similar to WARP, it can be explained using its phase noise figure. At 2.2 GHz, the vector signal generator [15] has a phase noise variance given by $\sigma_\phi = 0.06°$. The corresponding lower bound of the cancellation is ≈55 dB. Thus, the lower bound due to phase noise is close to upper bound of cancellation due to dynamic range limitations of the VSA, thus there is no apparent variation in active cancellation with varying delay.

IV. Active Analog Cancellation

Result 1: The amount of active analog cancellation in [1, 2] limited by the inverse of phase noise variance.

We denote the phase noise in the upconvertors in the self-interference path and the cancelling path by $\phi_{si}(t)$ and $\phi_c(t)$. Since both $\phi_{si}(t)$ and $\phi_c(t)$ are phase noises in the upconverting paths, therefore we assume $\mathbb{E}(|\phi_{si}(t)|^2) = \mathbb{E}(|\phi_c(t)|^2) = \sigma_{si}^2$. The phase noise at the receiver downconvertor is given by $\phi_d(t)$, whose variance is given by σ_d^2. The functions $\phi_{si}(t)$ can be correlated to $\phi_c(t)$, but are independent of $\phi_d(t)$.

In Fig. 3, we show a block diagram representation of the active analog canceller used in [1, 2]. The impulse response of the over-the-air channel is $h_{si}\delta(t-\Delta_{si})$. In [1, 2], the cancelling signal is generated by processing the self-interference signal in baseband prior to upconversion, which is captured by the function $f(.)$ shown in Fig. 3. Note that the local oscillator in the cancelling path is independent of the local oscillator in self-interference path, as is the case in [1, 2], thus the phase noises $\phi_{si}(t)$ and $\phi_c(t)$ are independent. The objective of the active analog canceller is to create a null for the self-interference. If the cancelling filter $f(t) = -h_{si}e^{-j\omega_c\Delta_{si}}\delta(t-\Delta_{si})$, then the cancelling signal is $x_c(t) = -h_{si}e^{-j\omega_c\Delta_{si}}x_{si}(t-\Delta_{si})$. After upconversion, the cancelling signal is given by $x_c(t)e^{j(\omega_c t+\phi_c(t))}$, and once the cancelling signal is added to the received self-interference signal, then the residual self-interference is given by

$$y_{res}(t) = h_{si}e^{j\omega_c(t-\Delta_{si})}x_{si}(t-\Delta_{si})(e^{j\phi_{si}(t-\Delta_{si})} - e^{j\phi_c(t)}) + z(t). \quad (11)$$

Note that, in the absence of phase noise, the residual self-interference in (11) will be only due to receiver thermal noise, which would imply a perfect null for the self-interference signal. However, the presence of phase noise results in an imperfect null. The strength of the residual self-interference signal can be estimated as

$$\begin{aligned} & \mathbb{E}(|y_{res}|^2) \\ = \; & \mathbb{E}\left(|h_{si}e^{j\omega_c(t-\Delta_{si})}x_{si}(t-\Delta_{si})(e^{j\phi_{si}(t-\Delta_{si})} - e^{j\phi_c(t)})|^2\right) \\ & + \mathbb{E}(|z(t)|^2) \\ = \; & |h_{si}|^2\mathbb{E}(|e^{j\phi_{si}(t-\Delta_{si})} - e^{j\phi_c(t)})|^2) + \sigma_z^2 \\ \overset{(a)}{\approx} \; & |h_{si}|^2\mathbb{E}(|\phi_{si}(t-\Delta_{si}) - \phi_c(t)|^2) + \sigma_z^2 \\ \overset{(b)}{=} \; & 2|h_{si}|^2\sigma_{si}^2 + \sigma_z^2, \end{aligned} \quad (12)$$

where (a) holds because $\phi_{si}(t) << 1, \phi_c(t) << 1$, and (b) is true since the phase noise in self-interference path and cancelling path are independent. The amount of cancellation is the ratio of the strength of self-interference before cancellation to the strength of self-interference after cancellation which is $\frac{|h_{si}|^2}{2|h_{si}|^2\sigma_\phi^2+\sigma_z^2} \leq \frac{1}{2\sigma_\phi^2}$. Plugging in the variance of phase noise, $\sigma_\phi = 0.7°$, for the MAXIM 2829 radio used in WARP, we obtain an upper bound of cancellation of 35 dB, which is very close to the reported cancellation numbers in [1]. The amount of active analog cancellation as a function of phase noise variance is plotted in Fig. 4, which shows that reducing phase noise in local oscillator can significantly improve the amount of active analog cancellation.

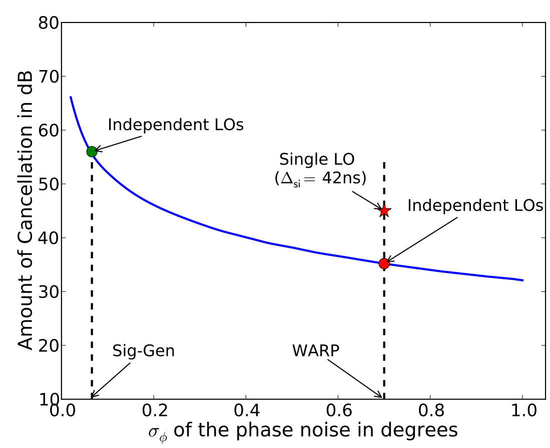

Fig. 4. Amount of active analog cancellation as a function of the variance of phase noise. The solid curve denotes the function $20\log_{10}(1/2\sigma_\phi^2)$

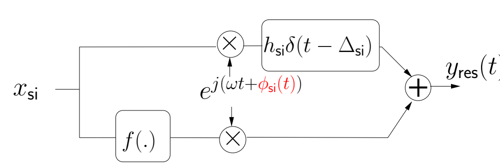

Fig. 5. Active analog canceller with matched local oscillator

In an alternate design, a single local oscillator can upconvert both the self-interference signal and the cancelling signal. The block diagram of an active analog canceller with a single local oscillator is shown in Fig. 5. Assuming perfect knowledge of the self-interference channel, the residual self-interference is

$$y_{\mathsf{res}}(t) \approx h_{\mathsf{si}} e^{j(\omega_c(t-\Delta_{\mathsf{si}}))} x_{\mathsf{si}}(t-\Delta_{\mathsf{si}})(\phi_{\mathsf{si}}(t-\Delta_{\mathsf{si}}) - \phi_{\mathsf{si}}(t)) + z(t) \tag{13}$$

and the strength of the residual self-interference is

$$\mathbb{E}(|y_{\mathsf{res}}(t)|^2) \approx 2|h_{\mathsf{si}}|^2 \sigma_{\mathsf{si}}^2 (1 - R_\phi(\Delta_{\mathsf{si}})) + \sigma_z^2. \tag{14}$$

In (13), the phase noises in the self-interference path and cancelling path go through different delays, thus resulting in an imperfect null for the self-interference signal. The amount of active analog cancellation can be bounded above as $\frac{|h_{\mathsf{si}}|^2}{2|h_{\mathsf{si}}|^2\sigma_{\mathsf{si}}^2(1-R_\phi(\Delta_{\mathsf{si}}))+\sigma_z^2} \leq \frac{1}{2\sigma_{\mathsf{si}}^2(1-R_{\phi_{\mathsf{si}}}(\Delta_{\mathsf{si}}))}$. The temporal correlation in phase noise aids in reducing the strength of the residual, thereby increasing the amount of cancellation compared to the case when the local oscillators are independent. To plot the amount of active analog cancellation possible, if a single local oscillator is used, we choose a representative delay of $\Delta_{\mathsf{si}} = 42$ns and MAXIM 2829 radio in WARP as the tranciever. From the measurements in Section III, we plot and observe in Fig. 4 that use of a single local oscillator for upconverting both self-interference and cancelling signal can increase the amount of cancellation by 10 dB.

V. Digital Cancellation

Result 2: The sum total of the amount of cancellation, for serially concatenated active analog and digital stages of cancellation, is bounded above by the inverse of the variance of phase noise of the local oscillators.

For brevity, we will restrict the discussion to designs of active analog cancellers with independent local oscillators in self-interference and cancelling paths. However, the arguements easily extend to designs with a single local oscillator. The digital canceller operartes on the residual self-interference after active analog cancellation. If the active analog canceller has perfect knowledge of the self-interference channel, then the residual self-interference after active analog cancellation is given by (11), which after downconversion can be approximted as

$$y_{\mathsf{res}}[iT] \approx h_{\mathsf{si}} e^{-j\omega\Delta_{\mathsf{si}}} x_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}](\phi_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}] - \phi_{\mathsf{c}}[iT]) e^{j(\phi_{\mathsf{c}}[iT]-\phi_{\mathsf{d}}[iT])} + z[iT]. \tag{15}$$

From (15), we note that the residual self-interference is a noisy version of the self-interference signal multiplied with function of phase noise. Since phase noise is unknown and changes every sample, $y_{\mathsf{res}}[iT]$ can be seen as a fast-fading version of the self-interference signal. The correlation of $y_{\mathsf{res}}[iT]$, with the self-interference signal itself is approximately zero because phase noise is zero mean and independent of the self-interference signal. Thus, if perfect knowledge of self-interference channel is available, then after active analog cancellation the residual self-interference signal is not correlated to the self-interference signal and hence the digital canceller does not reduce the self-interference at all.

Digital cancellation will help cancel self-interference only when the self-interference channel is not known perfectly. As an example, let us assume that there is a slight error in the knowledge of the delay of the self-interference channel. Let $\widehat{\mathbf{h}}_{\mathsf{si}}(t) = h_{\mathsf{si}}\delta(t-\tau)$, where $(\Delta_{\mathsf{si}} - \tau)$ denotes the error in the knowledge of delay. Then, the residual self-interference in digital baseband is given by

$$y_{\mathsf{res}}[iT] \approx h_{\mathsf{si}} e^{-j(\omega\Delta_{\mathsf{si}}+\phi_{\mathsf{c}}[iT]-\phi_{\mathsf{d}}[iT])}(x_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}] - x_{\mathsf{si}}[iT-\tau] + x_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}](\phi_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}] - \phi_{\mathsf{c}}[iT])) + z[iT].$$

The strength of the residual self-interference before digital cancellation is $2|h_{\mathsf{si}}|^2(1 - R_x(\Delta_{\mathsf{si}} - \tau) + \sigma_\phi^2) + \sigma_z^2$. The digital canceller can reduce the self-interference by using $-h_{\mathsf{si}} e^{-j\omega\Delta_{\mathsf{si}}}(x_{\mathsf{si}}[iT-\Delta_{\mathsf{si}}] - x_{\mathsf{si}}[iT-\tau])$ as the cancelling signal. After digital cancellation, the strength of residual self-interference is $2|h_{\mathsf{si}}|^2((1 - R_x(\Delta_{\mathsf{si}} - \tau))(\sigma_{\mathsf{si}}^2 + \sigma_{\mathsf{d}}^2) + \sigma_\phi^2) + \sigma_z^2$. Since the variance of phase noise is much smaller than unity, the digital canceller can reduce the strength of the residual self-interference by choosing appropriate cancelling signal. However, note that even after digital cancellation the strength of the residual self-interference is lower bounded by $2|h_{\mathsf{si}}|^2\sigma_{\mathsf{si}}^2 + \sigma_z^2$, which we recall is the strength of residual self-interference signal if the estimation error is zero.

Digital cancellation manages to cancel only the part of the residual self-interference which is correlated to the self-interference signal itself. The sum total of active analog and digital cancellation is upper bounded by $\frac{1}{2\sigma_{\mathsf{si}}^2}$. Thus, as the amount of active analog cancellation increases, the amount of digital cancellation deteriorates.

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